MULTIPLE CHOICE Circle the correct answers. **SCORE: _____ / 4 PTS**

[1] Which of these is/are improper ? #1➡ $\int_0^{\frac{\pi}{2}} \tan^{-1} x\, dx$ #2➡ $\int_{\frac{4\pi}{3}}^{\frac{2\pi}{3}} \csc x\, dx$ #3➡ $\int_1^3 \frac{1}{x^2-5x-6}\, dx$

CIRCLE THE LETTER BELOW WHICH CORRESPONDS TO THE CORRECT ANSWER.

[A]	none	[B]	only #1	[C]	only #2	[D]	only #3
[E]	only #1 and #2	[F]	only #1 and #3	[G]	only #2 and #3	[H]	all

2

[2] Which of these converge(s) ? #1➡ $\int_1^\infty \frac{1}{\sqrt{x}}\, dx$ #2➡ $\int_0^1 \frac{1}{x^\pi}\, dx$ #3➡ $\int_{-\infty}^0 \frac{1}{\pi^x}\, dx$

CIRCLE THE LETTER BELOW WHICH CORRESPONDS TO THE CORRECT ANSWER.

[A]	none	[B]	only #1	[C]	only #2	[D]	only #3
[E]	only #1 and #2	[F]	only #1 and #3	[G]	only #2 and #3	[H]	all

2

Give **very** brief answers. Explanations are **not** required. **SCORE: _____ / 4 PTS**

[1] If $h(x) \le x^{-3}$ for all x, what can you conclude about $\int_1^\infty h(x)\, dx$? NOTHING 2

[2] If $f(x) \ge g(x) \ge 0$ for all x, and $\int_0^\infty g(x)\, dx$ converges, what can you conclude about $\int_0^\infty f(x)\, dx$? NOTHING 2

Evaluate $\int_{-\infty}^1 xe^{2x}\, dx$. If the integral diverges, write "DIVERGES". **SCORE: _____ / 6 PTS**

$= \lim_{N\to-\infty} \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)\Big|_N^1$ 1½

$= \lim_{N\to-\infty} \left(\frac{1}{2}e^2 - \frac{1}{4}e^2 - \left(\frac{1}{2}Ne^{2N} - \frac{1}{4}e^{2N}\right)\right)$ 1½

$= \frac{1}{4}e^2 - (0-0)$

$= \frac{1}{4}e^2$ 1½

u	dv
x	e^{2x}
1	$\frac{1}{2}e^{2x}$
0	$\frac{1}{4}e^{2x}$

1½

$\lim_{N\to-\infty} Ne^{2N} = \lim_{N\to-\infty} \frac{N}{e^{-2N}} = \lim_{N\to-\infty} \frac{1}{-2e^{-2N}} = 0$

Evaluate $\int 9x^2 \arctan 3x \, dx$.

SCORE: ____ / 5 PTS

$= 3x^3 \arctan 3x - \int \frac{9x^3}{1+9x^2} dx$ (1) (1)

$= 3x^3 \arctan 3x - \int \left(x - \frac{x}{1+9x^2}\right) dx$ (1)

$= 3x^3 \arctan 3x - \frac{1}{2}x^2 + \frac{1}{18}\ln(1+9x^2) + C$ (1) (1)

u	dv
$\arctan 3x$	$9x^2$
$\frac{3}{1+9x^2}$	$\int$ $3x^3$

$9x^2+1 \overline{)\,9x^3}$ $\quad x$

$9x^3 + x$

$-x$

Evaluate $\int \frac{3t+1}{(t+2)(t^2-t-6)} dt$.

SCORE: ____ / 5 PTS

$= \int \frac{3t+1}{(t+2)(t+2)(t-3)} dt$ (1)

$= \int \left(\frac{A}{t+2} + \frac{B}{(t+2)^2} + \frac{C}{t-3}\right) dt$

$A(t+2)(t-3) + B(t-3) + C(t+2)^2 = 3t+1$ (1)

$t=3$: $25C = 10 \rightarrow C = \frac{2}{5}$

$t=-2$: $-5B = -5 \rightarrow B = 1$

COEF of t^2: $A + C = 0 \rightarrow A = -\frac{2}{5}$

$= \int \left(\frac{-\frac{2}{5}}{t+2} + \frac{1}{(t+2)^2} + \frac{\frac{2}{5}}{t-3}\right) dt$ ($\frac{1}{2}$) ($\frac{1}{2}$) ($\frac{1}{2}$)

$= -\frac{2}{5}\ln|t+2| - \frac{1}{t+2} + \frac{2}{5}\ln|t-3| + C$ ($\frac{1}{2}$) ($\frac{1}{2}$) ($\frac{1}{2}$)

SANITY CHECK $t=2$: $\frac{7}{4(-4)} \stackrel{?}{=} \frac{\frac{-2}{5}}{4} + \frac{1}{16} + \frac{\frac{2}{5}}{-1} = -\frac{1}{10} + \frac{1}{16} - \frac{2}{5} = \frac{-8+5-32}{80} = \frac{-35}{80} = \frac{-7}{16}$ ✓

Evaluate $\int \frac{t-202}{(t+3)(t^2-4t+20)} dt$.

SCORE: ____ / 6 PTS

$= \int \left(\frac{A}{t+3} + \frac{B(2t-4)+C}{(t-2)^2+16}\right) dt$

$A[(t-2)^2+16] + B(2t-4)(t+3) + C(t+3) = t-202$ (1)

$t=-3$: $41A = -205 \rightarrow A = -5$

$t=2$: $16A + 5C = -200 \rightarrow 5C = -120 \rightarrow C = -24$

COEF of t^2: $A + 2B = 0 \rightarrow B = -\frac{1}{2}A = \frac{5}{2}$

$= \int \left(\frac{-5}{t+3} + \frac{\frac{5}{2}(2t-4) - 24}{(t-2)^2+16}\right) dt$ ($\frac{1}{2}$) (1) (1) ($\frac{1}{2}$)

$= -5\ln|t+3| + \frac{5}{2}\ln(t^2-4t+20) - 6\tan^{-1}\frac{t-2}{4} + C$ ($\frac{1}{2}$) ($\frac{1}{2}$) (1)

SANITY CHECK $t=-2$: $\frac{-51}{-204} \stackrel{?}{=} \frac{-5}{1} + \frac{\frac{5}{2}(-8)-24}{32}$

$\frac{-51}{8} \stackrel{?}{=} -5 - \frac{44}{32}$

$= -5 - \frac{11}{8} = -\frac{51}{8}$ ✓